Propositions as Sessions

Logical Foundations of Concurrent Computation

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About Us

Dan

- Assistant Professor, Fundamental Computing Group
- **Research Interests**: Semantics of programming languages, program verification, and concurrent separation logics

Jorge

- Associate Professor and Leader, Fundamental Computing Group
- **Research Interests**: Models and semantics of concurrency, type systems, relative expressiveness

This Course: Propositions as Sessions

This course concerns the logical foundations of concurrent computation.

- You may have heard about ‘**propositions**’, but what do we mean by ‘**sessions**’?
- In a nutshell, a session is a convenient way of structuring a series of related interactions between communicating programs.
- A disciplined of these structures is key to ensure program correctness.
  Not only: they have elegant logical foundations!

This Course: Propositions as Sessions

Plan:

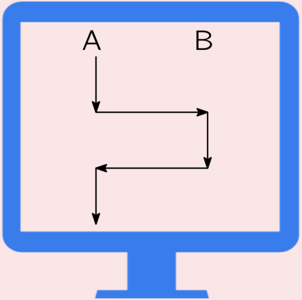
1. **Motivation** (Jorge) - Multiplicative Linear Logic (MLL) (Dan)
2. The concurrent interpretation of MALL (Jorge)
3. Cut-elimination and correctness for concurrent processes (Jorge)
4. Beyond linear resources: the !-modality and resource sharing (Dan)
5. An alternative view of resource sharing: Bunched Implications (Dan)

Your questions and feedback are warmly welcome!

Outline

When is a Program Correct?

Sequential Programs



"Programs produce outputs that are consistent with their input"

Concurrent Programs?

Message-Passing Concurrent Programs?

- Software components (**services**) distributed across networks

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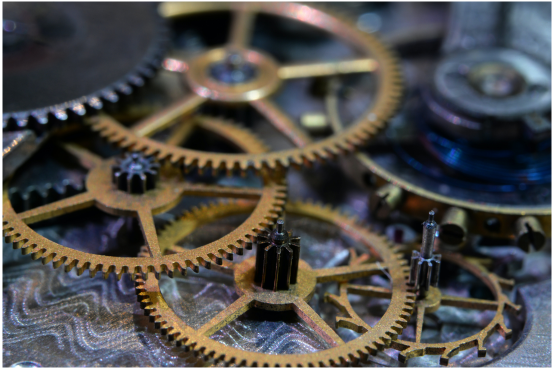
An (imperfect) analogy:



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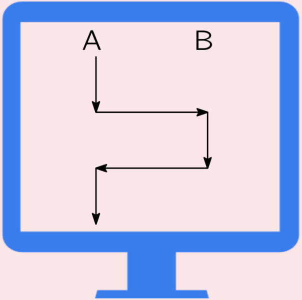

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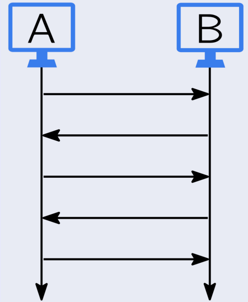

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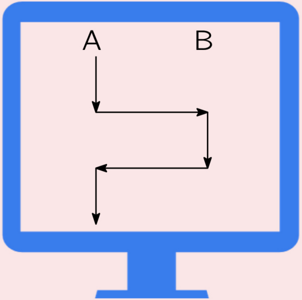

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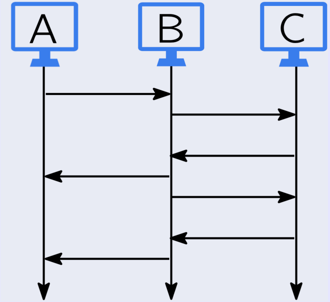

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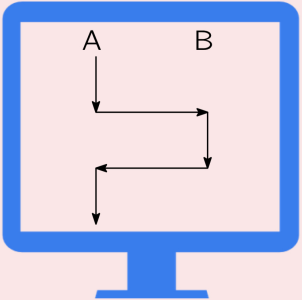

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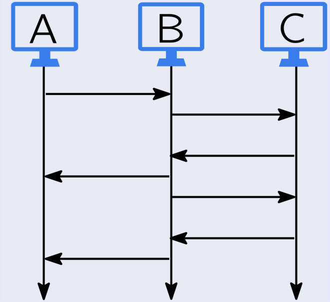

“Programs always respect their intended **protocols**”

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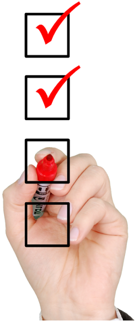
Note: The structure and sequentiality of messaging matters!

Example: A Two-Buyer Protocol

Desiderata for the implementations of Alice, Bob, and Seller:

- **Fidelity** – they **follow the intended protocol**.
  - Alice never ask Bob twice within the same conversation
  - Alice doesn't continue the transaction if Bob can't contribute
  - Alice chooses among the options provided by Seller



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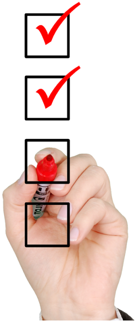
- **Safety** – they don't feature **communication errors**.
  - Seller always returns an integer when Alice requests a quote



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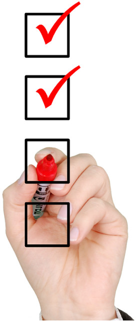
- **Deadlock-Freedom** – they do not "**get stuck**" while running the protocol.
  - Alice eventually receives an answer from Bob on his contribution.



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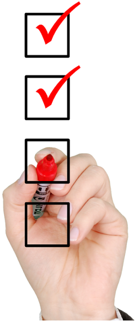

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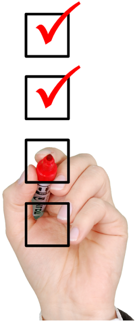

Correctness

- A **non-trivial notion**, which follows from the interplay of these properties.
- **Hard to enforce**, especially when actions are "scattered around" in programs.

$\rightarrow$ Sessions specify a protocol's structure, enabling program verification.
A session stipulates **what** and **when** should be exchanged (along a channel)

Type Systems

- Can detect bugs before programs are run
- Attached to many programming languages
- Implement a specific notion of correctness
  A program is either correct or incorrect

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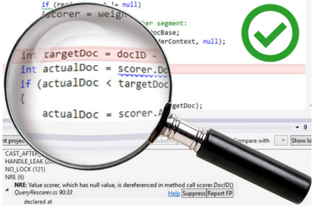

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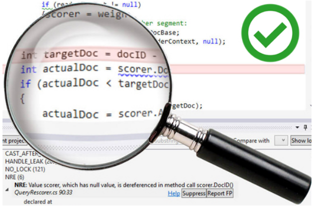

Sequential Languages

- **Data type systems** classify values in a program
- Examples: Integers, strings of characters

Concurrent Languages

- **Behavioral type systems** classify protocols in a program
- Example: “first send username, then receive true/false, finally close”
- A typical bug: sending messages in the wrong order

How to Design Type Systems? Logic to the Rescue!

-

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A landmark result in programming language theory:

- **Propositions as types** (Curry, 1935; Howard, 1969)

Propositions in Intuitionistic Logic	$\leftrightarrow$	Types
Proofs	$\leftrightarrow$	Sequential programs
Proof simplification	$\leftrightarrow$	Program evaluation

How to Design Type Systems? Logic to the Rescue!

- **Propositions as types** (Curry, 1935; Howard, 1969)

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- What about concurrent programs? A **resource-oriented** view!

How to Design Type Systems? Logic to the Rescue!

- **Propositions as sessions** (this course!)

Propositions in **Linear Logic** (LL)	$\leftrightarrow$	**Session types**
Proofs in LL	$\leftrightarrow$	Interacting processes
Cut elimination in LL	$\leftrightarrow$	process communication

Protocols as Session Types

Session types uniformly describe protocols in terms of

- communication actions (send and receive)
- choices (offers and selections)
- sequential composition
- recursion

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Session protocols are attached to **interaction devices**:

- communication channels in programs (think Go and Rust)
- TCP-IP sockets
- ...

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A formal syntax for protocols:

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Notice:

- Sequential communication patterns (no built-in concurrency)
- U stands for basic values (e.g. `int`) but also other sessions S

Example: A Two-Buyer Protocol

Recall the protocol between Alice, Bob, and Seller:

1. Alice sends a book title to Seller, who sends a quote back.
2. Alice checks whether Bob can contribute in buying the book.
3. Alice uses the answer from Bob (yes/no) to interact with Seller, either:
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Two independent protocols, with Alice “leading” the interactions:

1. A session type for Seller (in its interaction with Alice):

$$S_{\mathsf{SA}} = ?\mathtt{book};\, !\mathtt{quote};\, \& \begin{cases} \mathsf{buy}: & ?\mathtt{paym};\, ?\mathtt{address};\, !\mathtt{ok};\, \mathsf{end} \\ \mathsf{cancel}: & ?\mathtt{thanks};\, !\mathtt{bye};\, \mathsf{end} \end{cases}$$

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2. A session type for Alice (in its interaction with Bob):

$$S_{\mathsf{AB}} = !\texttt{cost};\ \&\begin{cases}\mathsf{share}: & ?\texttt{address};\ !\texttt{ok};\ \mathsf{end}\\ \mathsf{close}: & !\texttt{bye};\ \mathsf{end}\end{cases}$$

Example: A Two-Buyer Protocol

Implementations for Alice, Bob, and Seller should be **compatible**.

- **Duality** relates session types with opposite behaviors.
- Intuitively, the dual of sending is receiving (and vice versa).
- Similarly, branching is the dual of selection (and vice versa)

The dual of S is written $\overline{S}$.

Example: A Two-Buyer Protocol

Example:

- Recall that S_{AB} describes Alice's viewpoint in her interaction with Bob:

$$S_{\text{AB}} = \,!\texttt{cost};\, \& \begin{cases} \text{share}: & ?\texttt{address};\, !\texttt{ok};\, \text{end} \\ \text{close}: & !\texttt{bye};\, \text{end} \end{cases}$$

- Given this, Bob's implementation should conform to $\overline{S_{\text{AB}}}$, the dual of S_{AB}:

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- Also, Alice's implementation should conform to both $\overline{S_{\mathsf{SA}}}$ and S_{AB}.

Session Type Duality, Formally

Given a (finite) session type S, its dual type $\overline{S}$ is inductively defined as follows:

$$
\begin{aligned}
\overline{!\,U;S} &= ?\,U;\overline{S} \\
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\overline{\&\{l_i : S_i\}_{i\in I}} &= \oplus\{l_i : \overline{S_i}\}_{i\in I} \\
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\overline{\mathsf{end}} &= \mathsf{end}
\end{aligned}
$$

Notice:

- Duality for recursive session types is defined coinductively (the dual of $\mu t.S$ is *not* $\mu t.\overline{S}$)

Taking Stock

Up to here:

- Correctness for communicating programs
- Sessions as protocol specifications
- A formal syntax for session types
- Example: A two-buyer protocol
- The notion of duality for session types

Coming next:

- Linear logic, in its intuitionistic variant